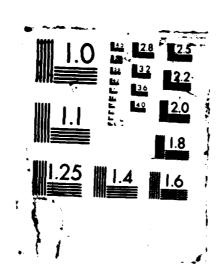
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# AFOSR-TR- 87-1734

### VIBRATIONS OF STRUCTURES WITH PARAMETRIC UNCERTAINTIES

Haym Benaroya, Ph.D., Principal Investigator
Weidlinger Associates
333 Seventh Avenue
New York, New York 10001

Dr. Anthony Amos, Program Manager AFOSR Contract No F49620-84-C-0009

### PROBLEM STATEMENT AND MOTIVATION

The focus of this research effort is the study of structural dynamics with parameter uncertainties. The motivation for this study rests with the need to understand the dynamics and control of large space structures such as the Space Station. The parameter uncertainties include structural mass, damping, and stiffness, and environmental loading.

A basis for this study comes from the realization that numerous uncertainties exist in the modeling and analysis of large space structures. These include:

- 1. deployment of structural components; since it will be impossible to bring into orbit a complete space structure, components will be carried into orbit on the Space Shuttle, then deployed and constructed in a manner that depends on the specific component. Deployment dynamics in general will involve uncertain initial conditions and may depend on environmental factors such as solar radiation. If the structure to be deployed is of the truss type, then deployment dynamics (transient) must account for uncertainties in nodal coordinates and the properties (such as damping) of the joints.
- 2. loading uncertainties; loads on the space station will be due to either unpredicted sources or to known but very complex sources. These loads will be transient and best modeled as random processes. Examples of such loads are thermal stresses, dynamics of deployment and operation, and impulses due to impacts.
- 3. material properties; while it is expected that the properties of materials to be used in space station design will be well known, certain structural types, such as composites, may be best modeled in a probabilistic fashion. Furthermore, it is very important to consider and predict fatigue and fracture lives of Space Station components. These are inherently probabilistic problems.
- 4. extreme behavior; to understand the limits of Space Station behavior we must determine quantities such as extreme value statistics and thus able to establish confidence bounds on undesirable behavior.



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5. parameter estimation and system sensitivity; since much of our understanding of structural mechanics is derived from experimental studies, it is imperative that confidence bounds be placed on parameter values and that system sensitivity to parameter deviations be established.

#### TECHNICAL APPROACH

The engineering community has only recently become convinced of the necessity to account for uncertainties in analysis and design. The use of probabilistic modeling techniques and statistical data analysis has become an important area for engineering Research and Development. However, to date, only relatively simple structures can be analyzed where structural parameters and/or environmental loads have significant random components.

Various techniques have been attempted to analyze structures where significant uncertainties have been associated with mass, damping, or stiffness. These techniques were based on major simplifying assumptions. In the current effort, an "iterative/ decomposition" technique was adopted that permits the analyst to proceed without making initial assumptions or simplifications. In this manner, a most general approach can be attempted and insurmountable mathematical difficulties can be addressed as they appear in the analysis. Thus, needless simplifications may be avoided. The formal technique is not new, but its previous applications have been implemented with a priori simplifications and without numerical results for verification. The present effort has been worked through to numerical results which, by way of a Monte Carlo simulation, will be verified within prescribed error bounds.

#### SUMMARY OF RESULTS

We have modeled a simple structure with random process stiffness and environmental loads. Two physical situations which may be modeled in such a fashion are structures that have parametric loads (pin-ended column with axial random excitation), and situations where the structural material may decay in a random manner.

Two cases have so far been considered with the iterative/decomposition technique:

- i) non-white (colored) noise random processes, and
- ii) white-noise random processes.

The first is physically more realistic and much more difficult mathematically. This most general case was undertaken first and initial analytic/numerical (using symbolic manipulation code MACSYMA) results indicate that stiffness parameter uncertainties have a significant effect on the structural dynamics. Recently, by designing a Monte Carlo simulation model of the structure and environment, we have been able to estimate errors due to approximations in (i). These comparisons with simulation have permitted us to simplify the approximate technique by limiting the expansion to a one-term "correction" to account for system uncertainties.

The second case, with the assumption of white-noise random processes, is much simpler analytically, and, in fact, it is possible to obtain exact, closed-form solutions.

The implications from these results are the following:

- (i) parameter uncertainties can significantly alter structural response and, equally important, can result in unstable structural behavior; bounds for stable behavior have to be established where parametric excitation exists;
- (ii) ranges for parameter values can be established to assure not only stability, but also error bounds.

Additional studies were performed regarding stochastic stability; this has been accomplished in addition to contracted tasks because it had been determined that such work was needed to provide a more complete dimension to the research effort.

Since algebraic/computational codes such as MACSYMA are relatively new to such work, it was decided to gather our experience and collaborate with two academics (DiMaggio and Elishakoff) to issue a paper summarizing our work.

#### THIRD YEAR WORK

The Monte Carlo results make it possible to consider extensions of the model to more complicated, but specific, types of structures. In this manner, the analysis becomes less general when specific types of structures with more narrowly defined uncertainties are studied.

We have concentrated our third year efforts on problems of stochastic stability, stationarity of stochastic systems, extending our understanding of this expansion method to large systems by way of "stochastic finite element methodologies", and applying the method to structures.

Appendices are included which demonstrate the importance of incorporating structural uncertainties in such analyses. Appendix A is a group of figures where only the input is a random process. These curves demonstrate the correctness and effectiveness of the Monte Carlo simulations that have been performed for purposes of model verification. Appendix B is a group of curves where the system stiffness parameter K(t) is assumed to be positive-definite. Here, we check the limits of the derived approximate method as the variance of the stochastic parameter is increased to very large values (1). Appendix C is a group of figures that demonstrate the effects of system (K(t)) randomness on system response. Two facts to note are that (i) there can be a major difference in response variance, and (ii) there can be major differences in peak "energy", i.e., the peak Spectral Density value at system natural frequency  $(S_{xx}(\omega = 1))$  will differ by orders of magnitude. All these issues have been discussed in detail in our papers and reports.

On the next page is a list, by year, of our research reports, presentations, and published refereed papers.

## REPORTS ISSUED, PRESENTATIONS and PAPERS PUBLISHED

## Air Force Office of Scientific Research Contract No F49620-84-C-0009

#### Year 1: Jan-Dec 1984

"Stochastic Structural Dynamics - A Theoretical Basis and a Selective Review of the Literature," H Benaroya, Weidlinger Associates Technical Report No 1, April 1984

"Stochastic Differential Equations – A Selective Overview for Structural Dynamics, Applications," H Benaroya, Proceedings of the 25th AIAA-ASME-ASCE-AHS Structures, Structural Dynamics, and Materials Conference, Palm Springs, May 1984

"The Decomposition Method of Adomian – Its Application to Problems of Structural Dynamics," H Benaroya, Weidlinger Associates Technical Report No 2, May 1984

#### Year 2: Jan-Dec 1985

"Random Vibrations with Parametric Uncertainties," H Benaroya, M Rehak, Weidlinger Associates Technical Report No 3, January 1985, published as "Parametric Random Excitation. I: Exponentially Correlated Parameters", ASCE J Eng Mech, Vol 113, No 6, June 1987

"The Decomposition Method in Stochastic Structural Dynamics," H Benaroya, M Rehak, Proceedings, 26th AIAA-ASME-ASCE-AHS Structures, Structural Dynamics, and Materials Conference, Orlando, April 1985

"The Iterative Method for White Noise – Its Application to Problems of Structural Dynamics," H Benaroya, M Rehak, Weidlinger Associates Technical Report No 4, May 1985, published as "Parametric Random Excitation. II: White-Noise Parameters" ASCE J Eng Mech, Vol 113, No 6, June 1987

"Neumann Series Approach to Random Vibrations," H Benaroya, M Rehak, Proceedings Fifth VPI&SU/AIAA Symposium on Dynamics and Control of Large Structures, Blacksburg, June 1985

"An Iterative Method in Stochastic Structural Dynamics," H Benaroya, M Rehak, Presentation at AFOSR's Third Forum on Large Space Structures, Texas A&M, College Station, July 1985

## Year 3: Jan 1986 — Sept 1987

"Verification of the Iterative Method with Monte Carlo Simulation," M Rehak, H Benaroya, Weidlinger Associates Technical Report No 5, March 1986; Proceedings of the 27th AIAA-ASME-ASCE-AHS Structures, Structural Dynamics, and Materials Conference, San Antonio, May 1986; accepted for publication by AIAA J Guidance, Control, and Dynamics

"Response and Stability of a Random Differential Equation – Part I: Moment Equation Method," H Benaroya, M Rehak, Weidlinger Associates Technical Report No 6, April 1986; submitted for publication to Int J Eng Sci

"Response and Stability of a Random Differential Equation – Part II: Iterative Method," H Benaroya, M Rehak, Weidlinger Associates Technical Report No 7, May 1986; submitted for publication to Int J Eng Sci

"Random Vibrations with MACSYMA," M Rehak, F DiMaggio, H Benaroya, I Elishakoff, Weidlinger Associates Technical Report No 8, June 1986; presented to the First World Congress on Computational Mechanics, Austin, September 1986; Computer Meth in Appl Mech and Eng 61 (1987) 61-70

"Application of the Iterative Method to Structures," M Rehak, H Benaroya, Weidlinger Associates Technical Report No 9, September 1986; presented to the New Orleans Structures Congress, Sept 1986; presented to ASCE Fall Convention, Boston, October 1986; accepted for publication as "The Harmonic Oscillator with Random Stiffness" to Mechanics of Structures and Machines

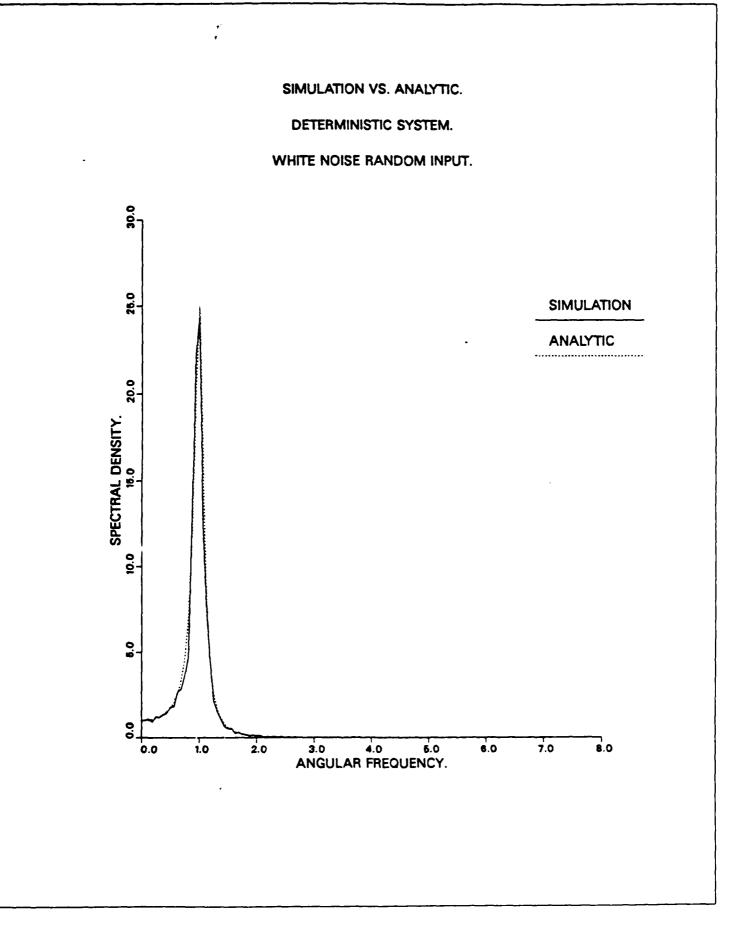
"Neumann Series/Born Approximation Applied to Parametrically Excited Stochastic Systems," H Benaroya, M Rehak, J of Prob Eng Mech 1987 Vol 2, No 2

"Non-Recursive Statistics for Integral Equation Solutions," H Benaroya, accepted for publication by Appl Math and Computation

"Finite Element Methods and Probabilistic Structural Analysis — A Selective Review," H Benaroya, M Rehak, submitted for publication to Applied Mechanics Reviews

"Stationarity and Stochastic Differential Equations," M Rehak, H Benaroya, submitted for publication to Int J Eng Sci

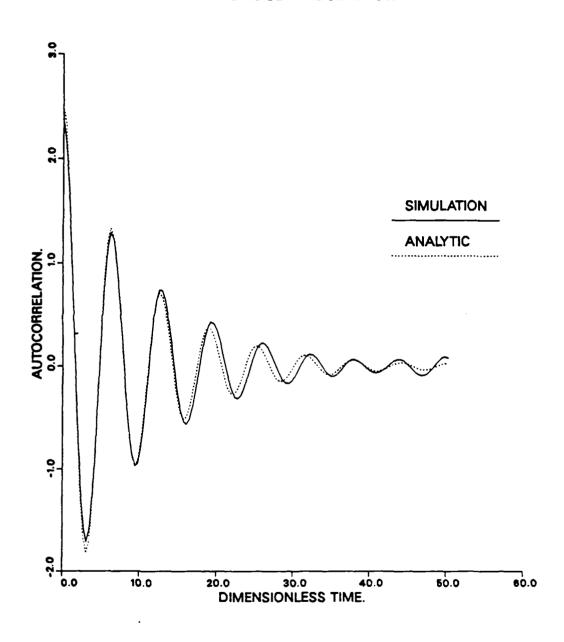
## APPENDIX A

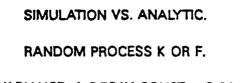


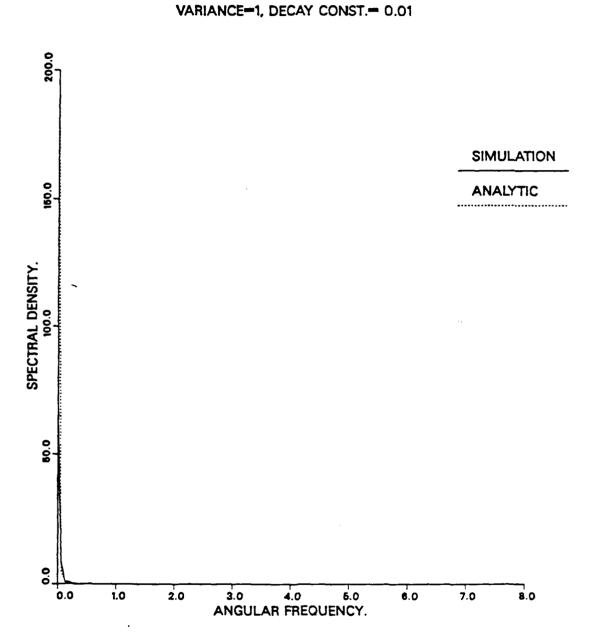


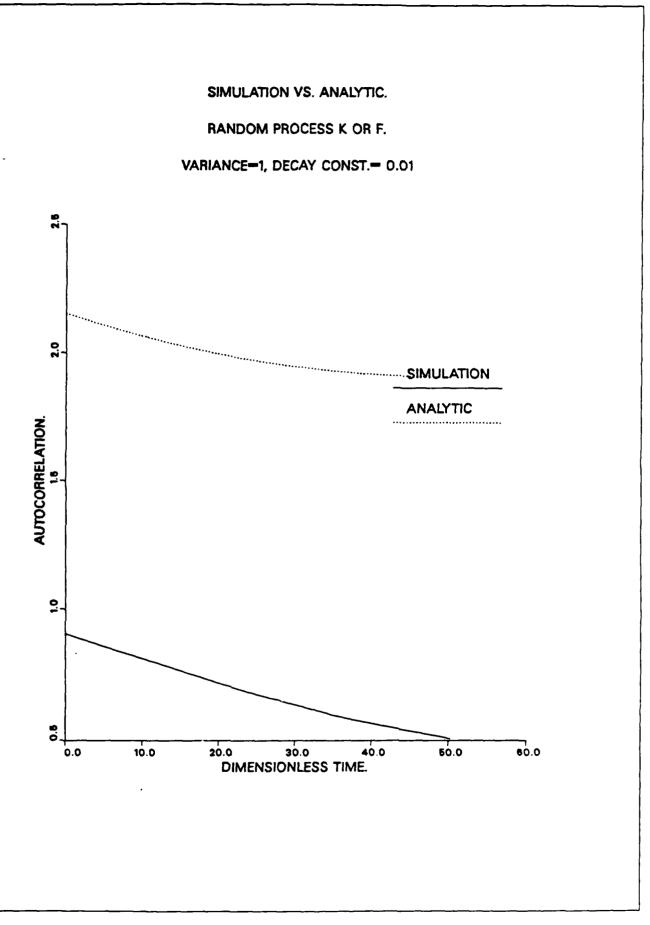
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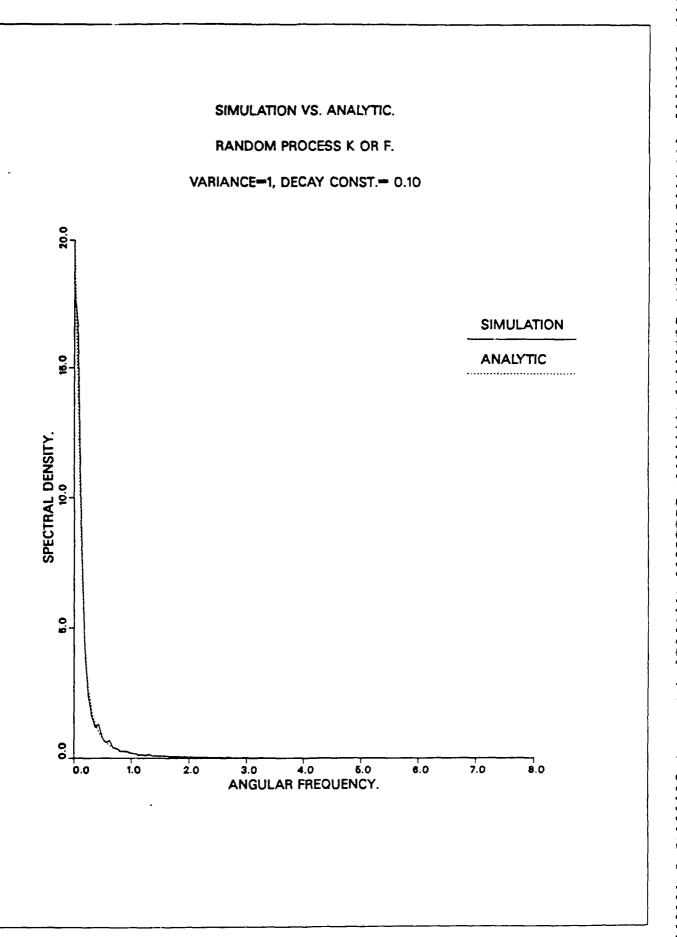
## WHITE NOISE RANDOM INPUT.

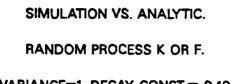


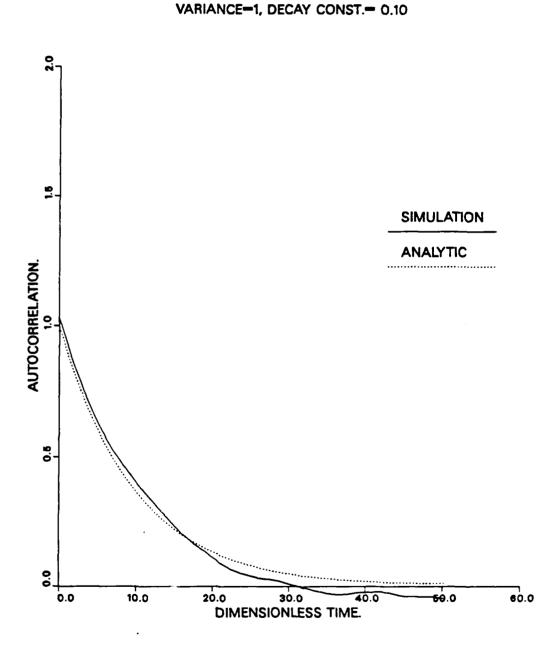


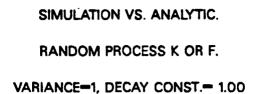


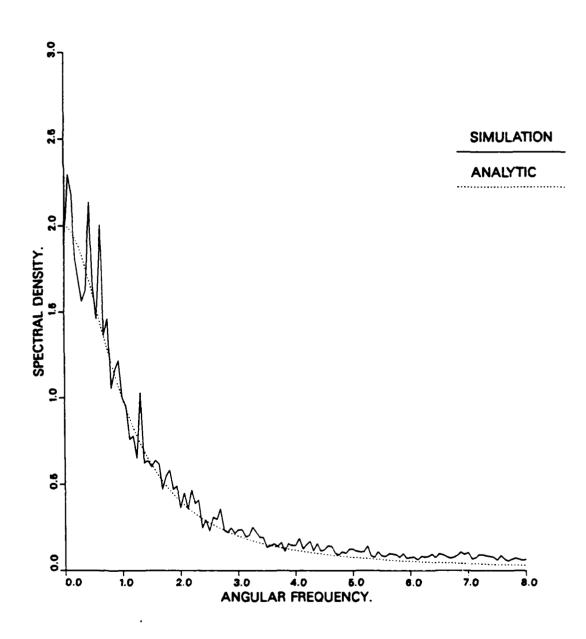


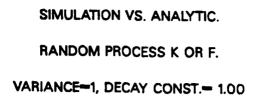


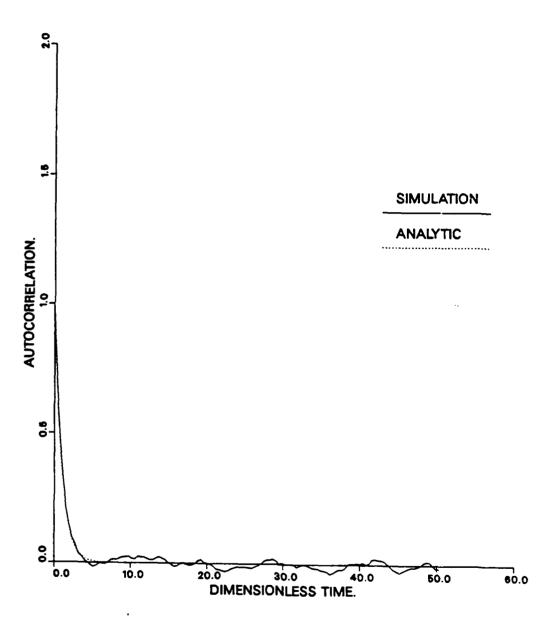


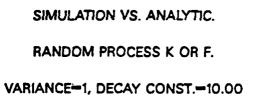


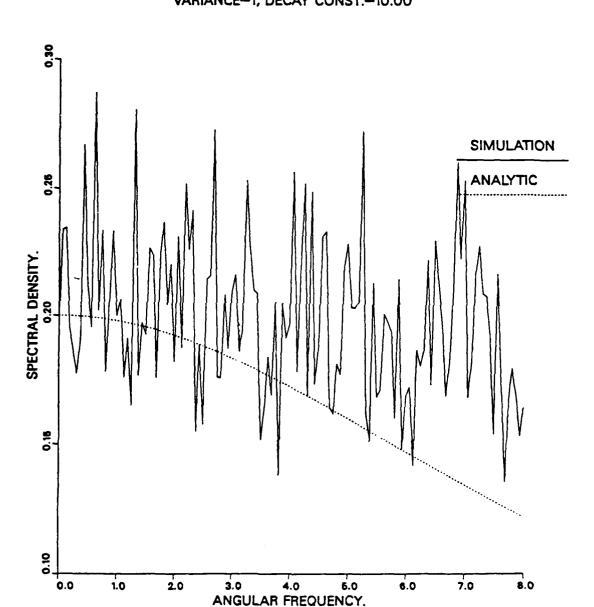


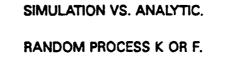




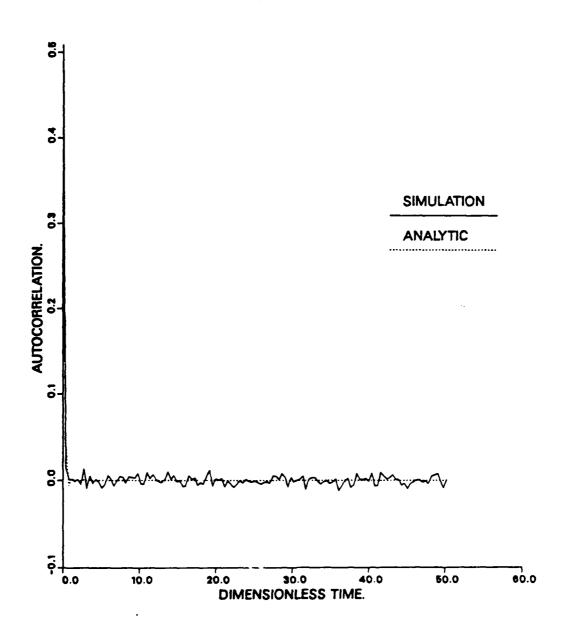












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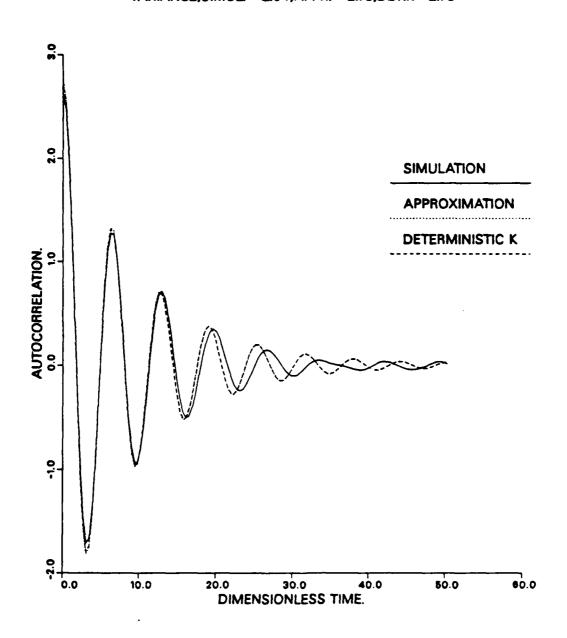
## APPENDIX B



SIMULATION VS. APPROXIMATION.

VAR.OF K= 0.00,BET= 0.01,ALPH= 1.00,Z= 0.10

VARIANCE, SIMUL = 2.64, APPR. = 2.75, DET. K = 2.73

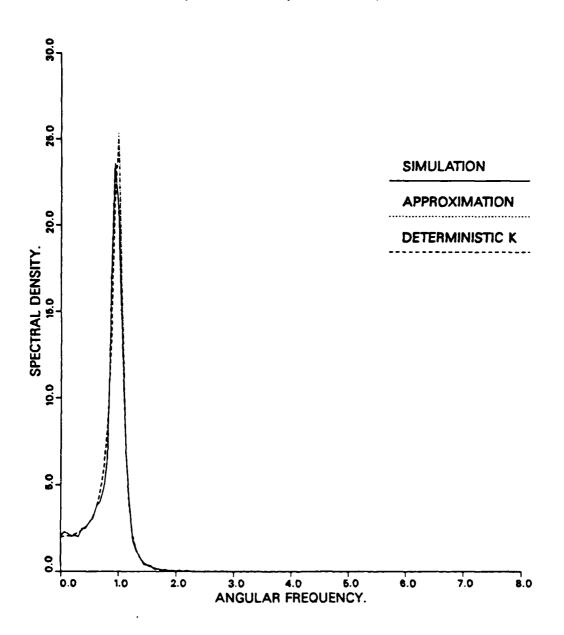


#### POSITIVE RANDOM STIFFNESS

### SIMULATION VS. APPROXIMATION.

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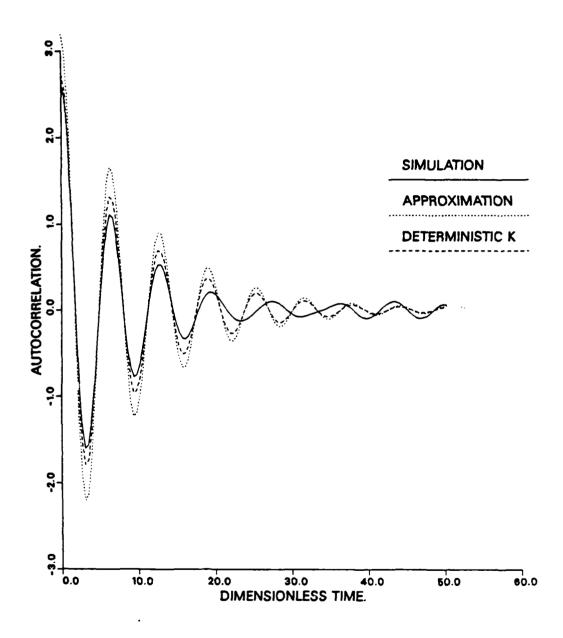
PEAK AT 1,SIMUL=20.84 ,APPR.=25.33,DET.K=25.00





VAR.OF K= 0.02,BET= 0.10,ALPH= 1.00,Z= 0.10

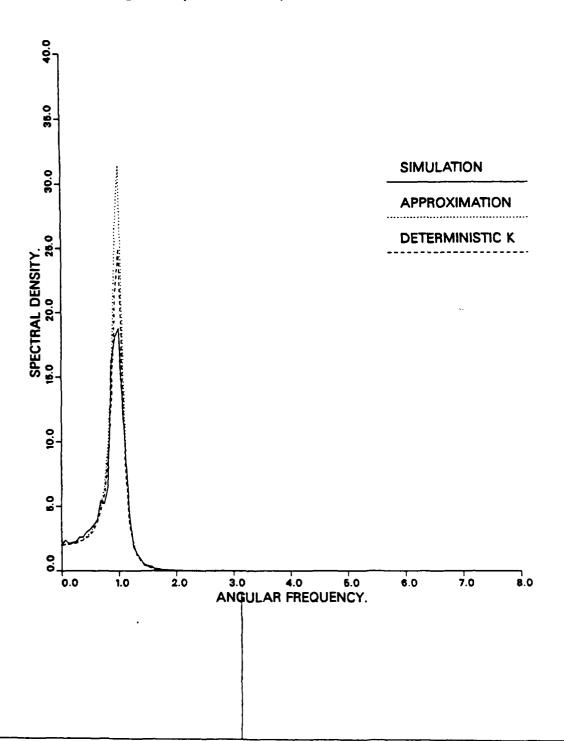
VARIANCE, SIMUL = 2.61, APPR. = 3.19, DET. K = 2.73



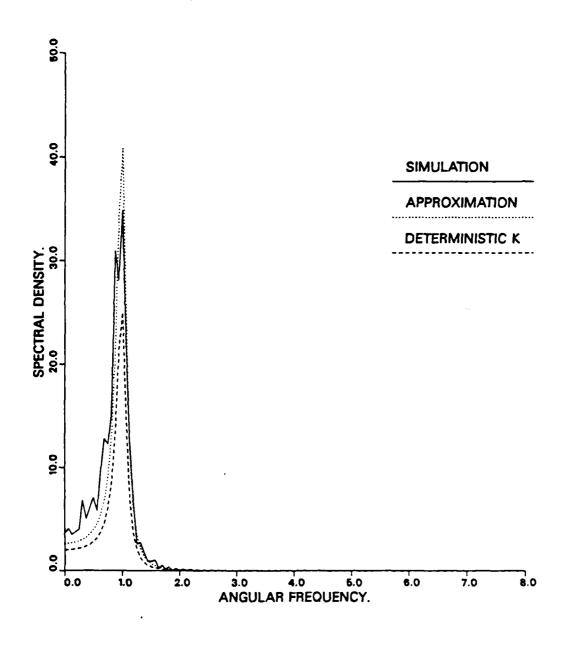


VAR.OF K= 0.02,BET= 0.10,ALPH= 1.00,Z= 0.10

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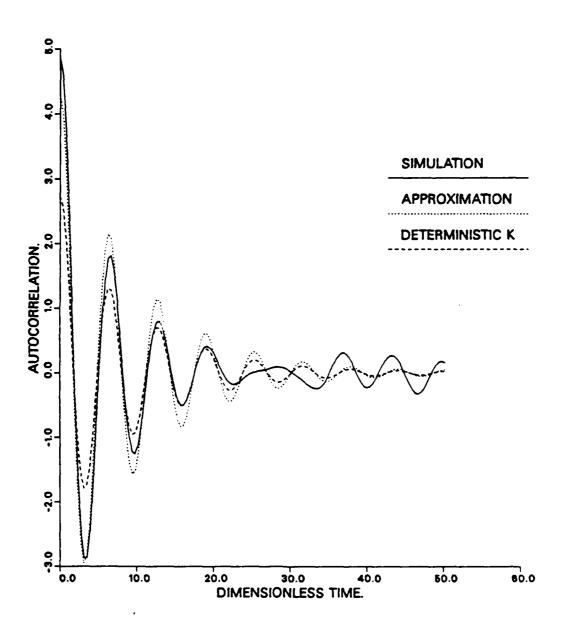




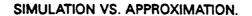


VAR.OF K= 0.20,BET= 1.00,ALPH= 1.00,Z= 0.10

VARIANCE, SIMUL. = 4.87, APPR. = 4.29, DET. K = 2.73

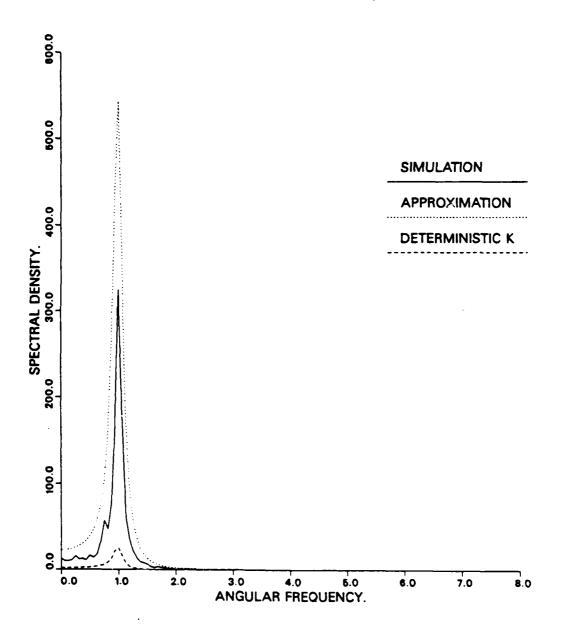


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VAR.OF K= 0.38,BET=10.00,ALPH= 1.00,Z= 0.10

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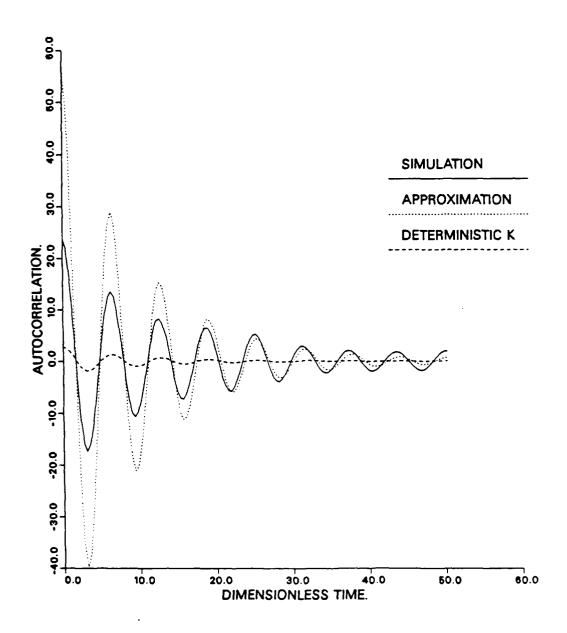


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VAR.OF K= 0.38,BET=10.00,ALPH= 1.00,Z= 0.10

VARIANCE, SIMUL. = 23.59, APPR. = 54.54, DET. K = 2.73

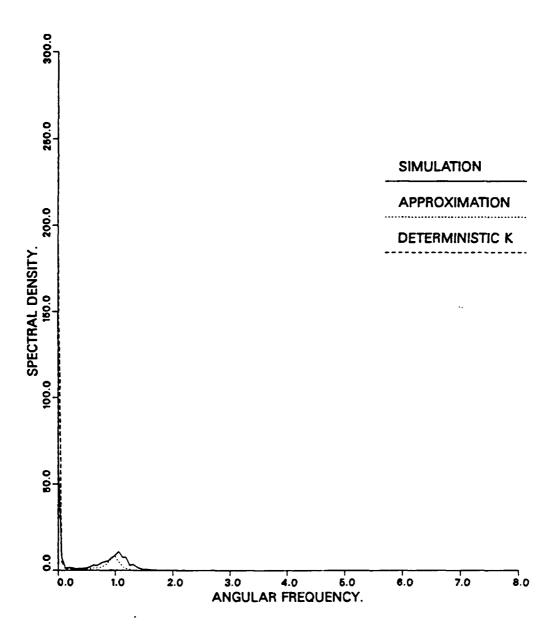




SIMULATION VS. APPROXIMATION.

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PEAK AT 1,SIMUL= 8.75 ,APPR.= 8.18,DET.K= 0.50

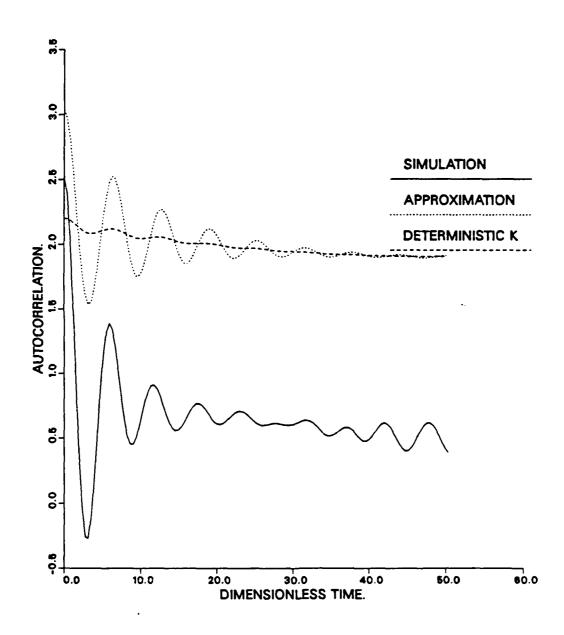




SIMULATION VS. APPROXIMATION.

VAR.OF K= 1.00,BET= 1.00,ALPH= 0.01,Z= 0.10

VARIANCE, SIMUL = 2.53, APPR. = 3.03, DET. K = 2.20

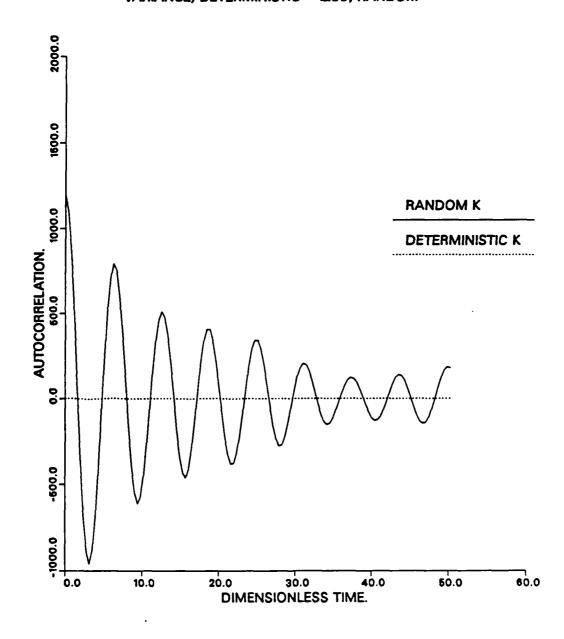


## APPENDIX C

RANDOM VS. DETERMINISTIC SYSTEM.

VARIANCE OF K = 0.50, BETA =10.00, ALPHA =10.00

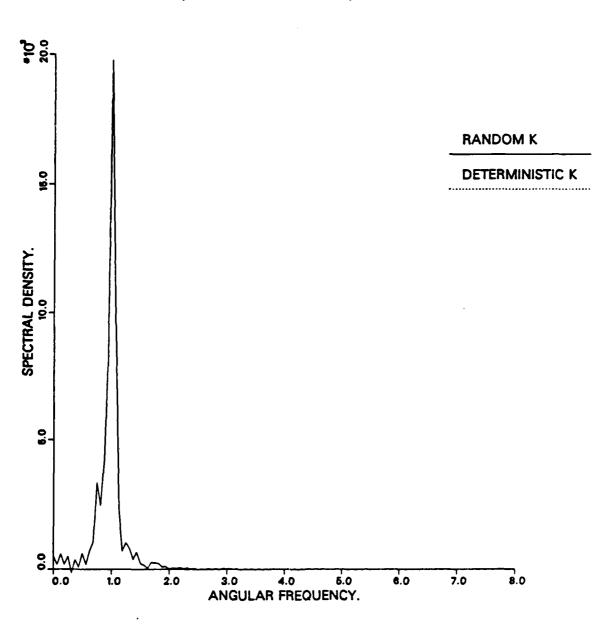
VARIANCE, DETERMINISTIC = 2.50, RANDOM =\*\*\*\*\*

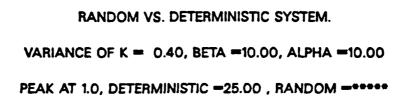


RANDOM VS. DETERMINISTIC SYSTEM.

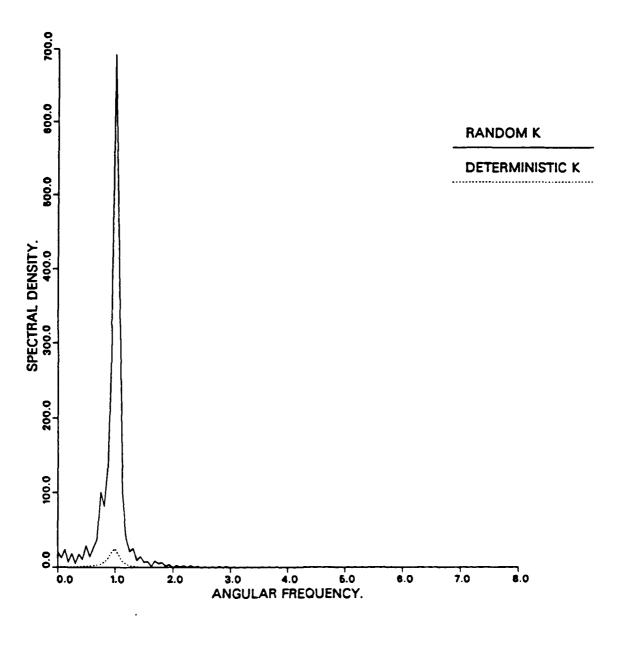
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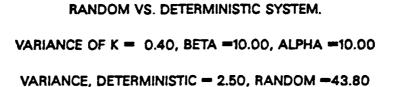
PEAK AT 1.0, DETERMINISTIC =25.00, RANDOM =\*\*\*\*\*

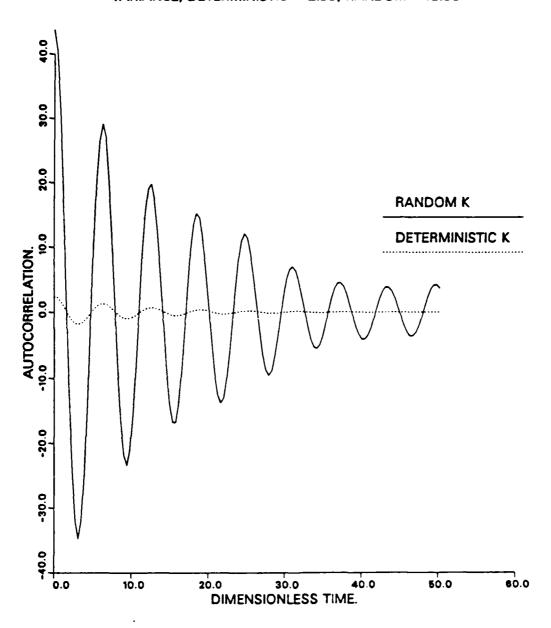




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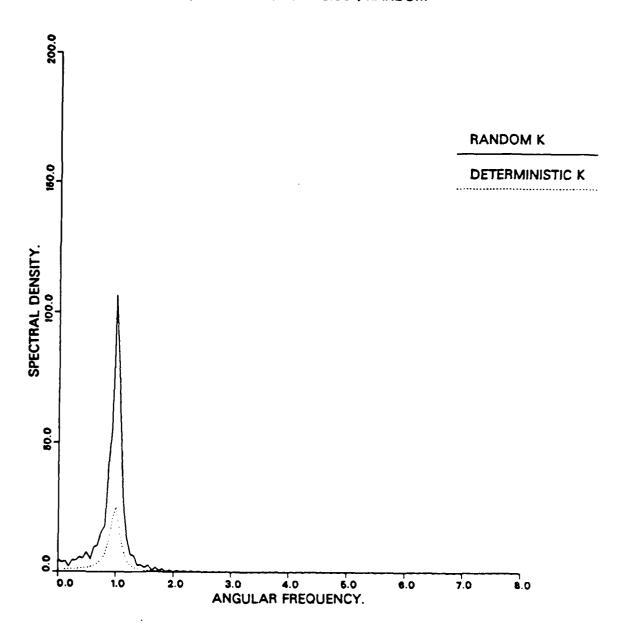






VARIANCE OF K = 0.30, BETA =10.00, ALPHA =10.00

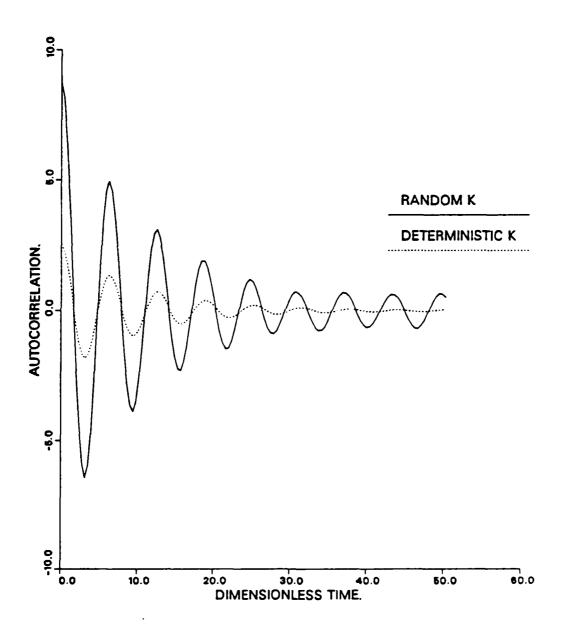
PEAK AT 1.0, DETERMINISTIC =25.00 , RANDOM =\*\*\*\*\*





VARIANCE OF K = 0.30, BETA =10.00, ALPHA =10.00

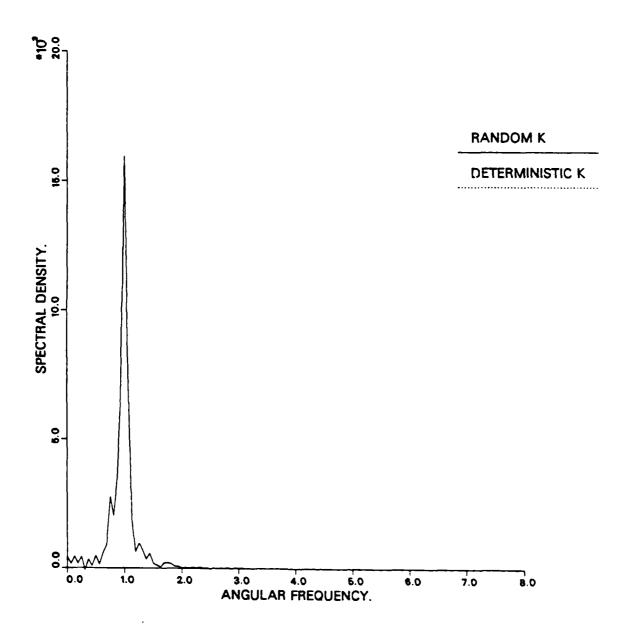
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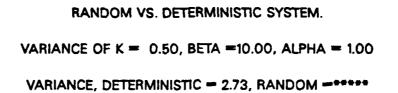


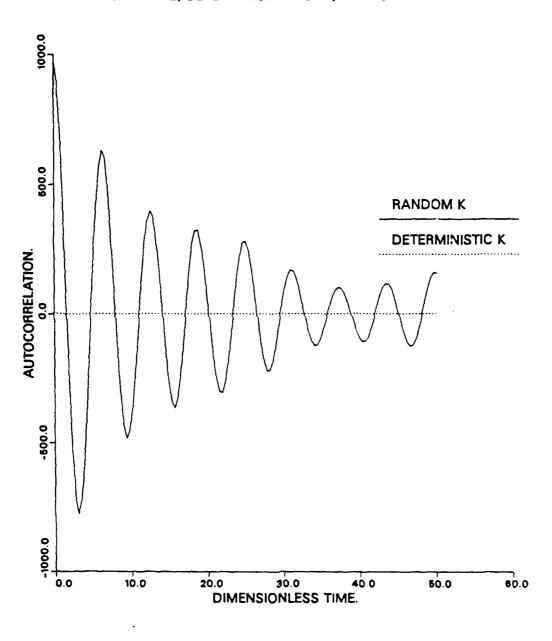


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PEAK AT 1.0, DETERMINISTIC =25.00 , RANDOM =\*\*\*\*\*



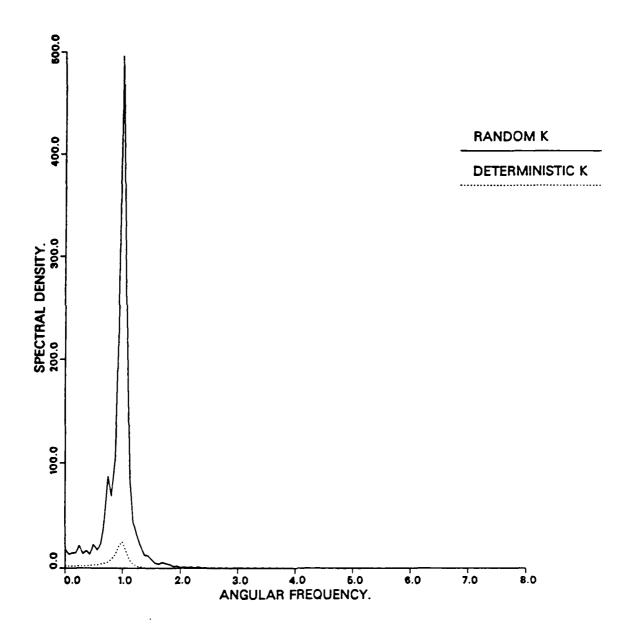


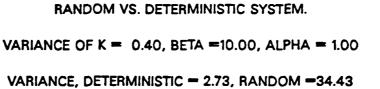


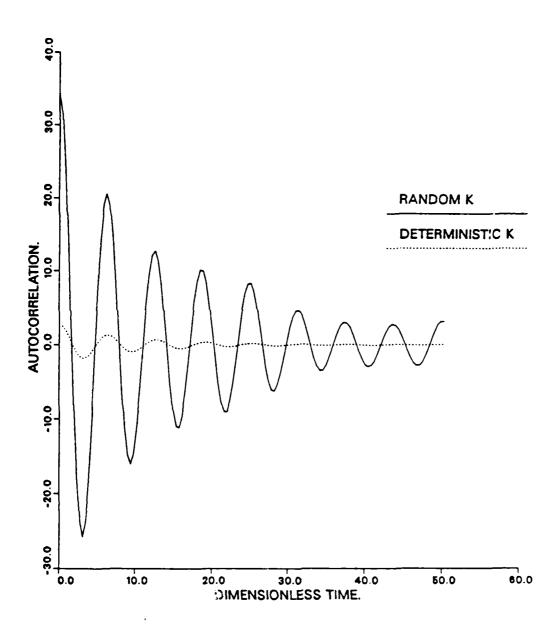


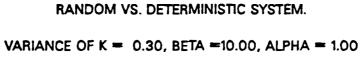
VARIANCE OF K = 0.40, BETA = 10.00, ALPHA = 1.00

PEAK AT 1.0, DETERMINISTIC =25.00, RANDOM =\*\*\*\*\*

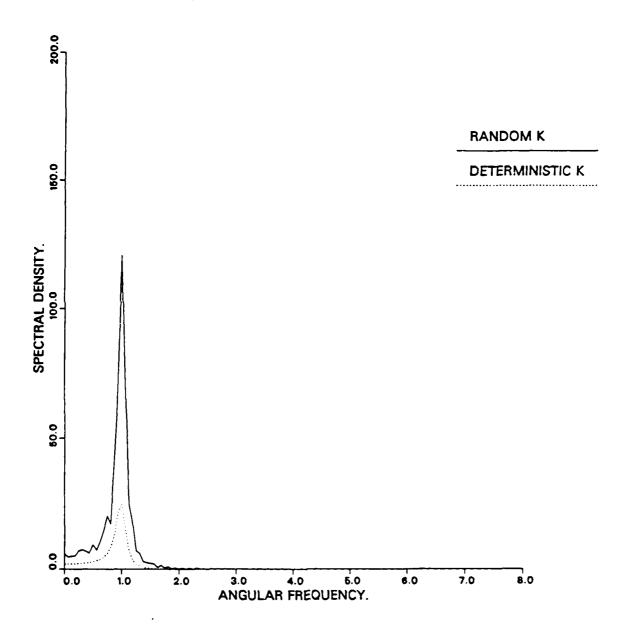








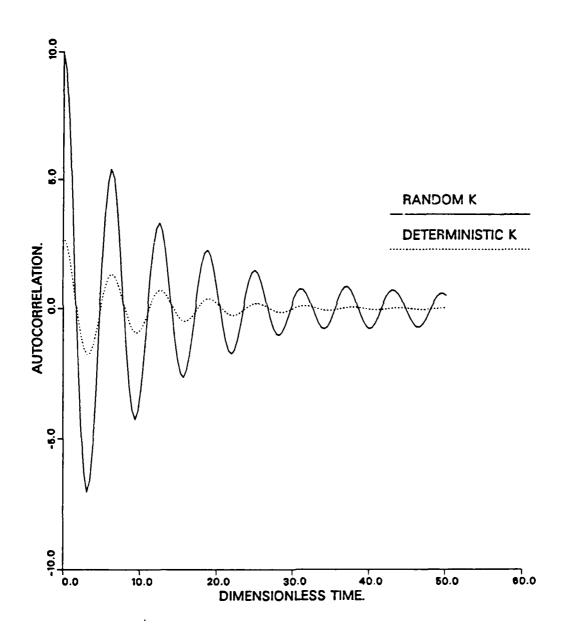
PEAK AT 1.0, DETERMINISTIC =25.00, RANDOM =\*\*\*\*\*





VARIANCE OF K = 0.30, BETA =10.00, ALPHA = 1.00

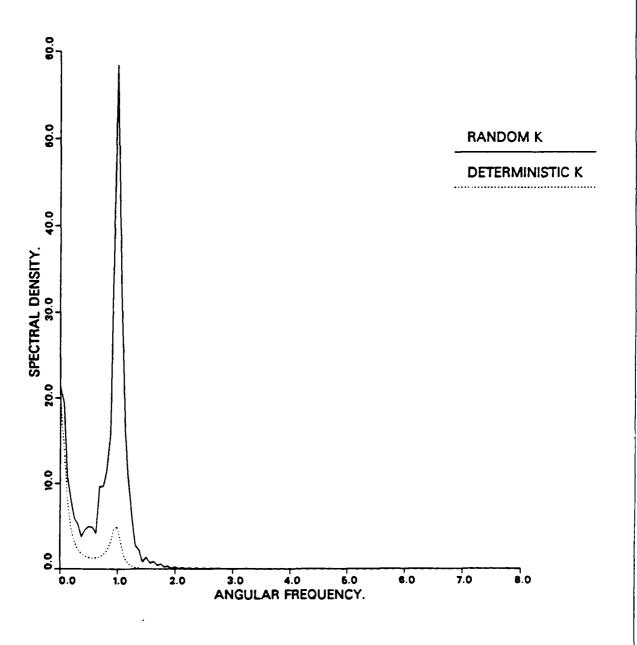
VARIANCE, DETERMINISTIC = 2.73, RANDOM = 9.98





VARIANCE OF K = 0.30, BETA =10.00, ALPHA = 0.10

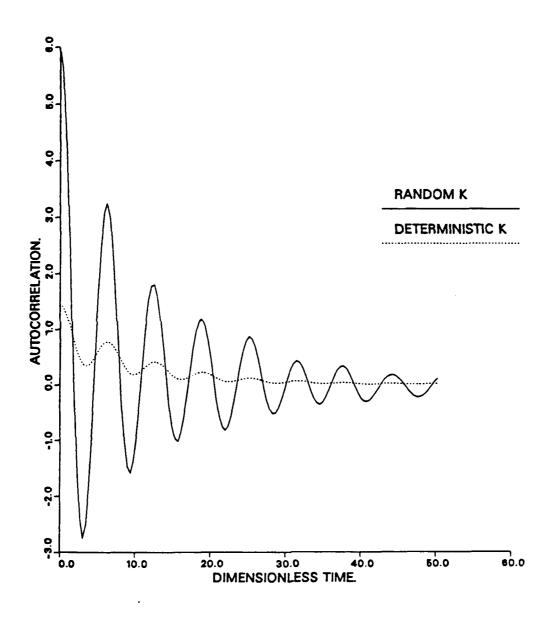
PEAK AT 1.0, DETERMINISTIC = 4.95, RANDOM =58.35





VARIANCE OF K = 0.30, BETA =10.00, ALPHA = 0.10

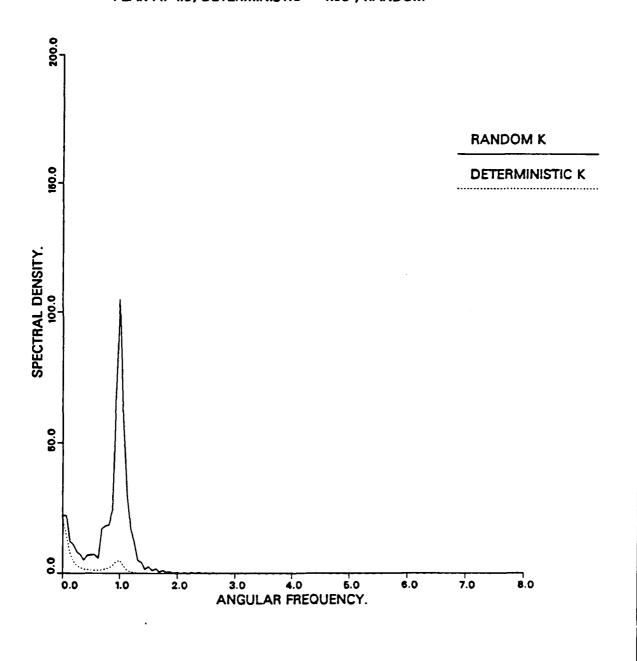
VARIANCE, DETERMINISTIC = 1.46, RANDOM = 5.99



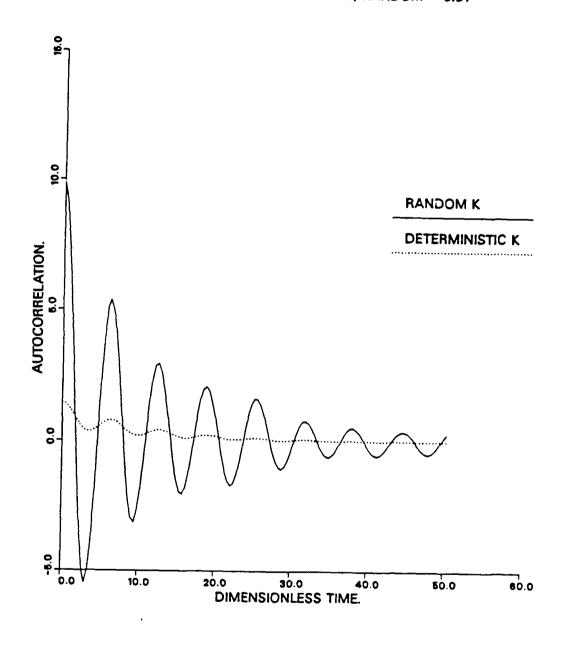


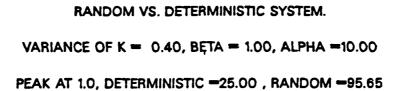
VARIANCE OF K = 0.35, BETA =10.00, ALPHA = 0.10

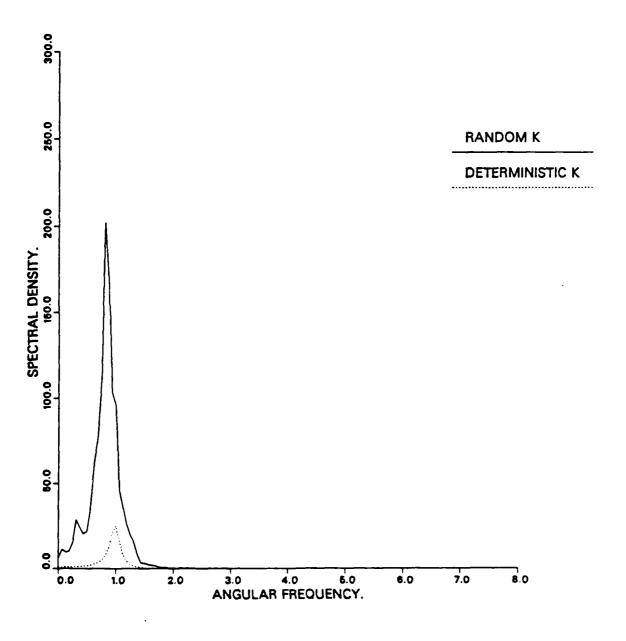
PEAK AT 1.0, DETERMINISTIC = 4.95, RANDOM =\*\*\*\*\*



RANDOM VS. DETERMINISTIC SYSTEM. VARIANCE OF K = 0.35, BETA =10.00, ALPHA = 0.10 VARIANCE, DETERMINISTIC = 1.46, RANDOM = 9.81



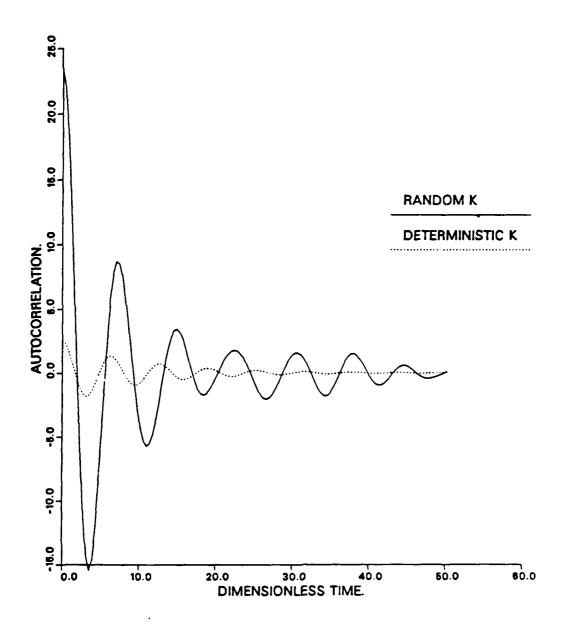






VARIANCE OF K = 0.40, BETA = 1.00, ALPHA = 10.00

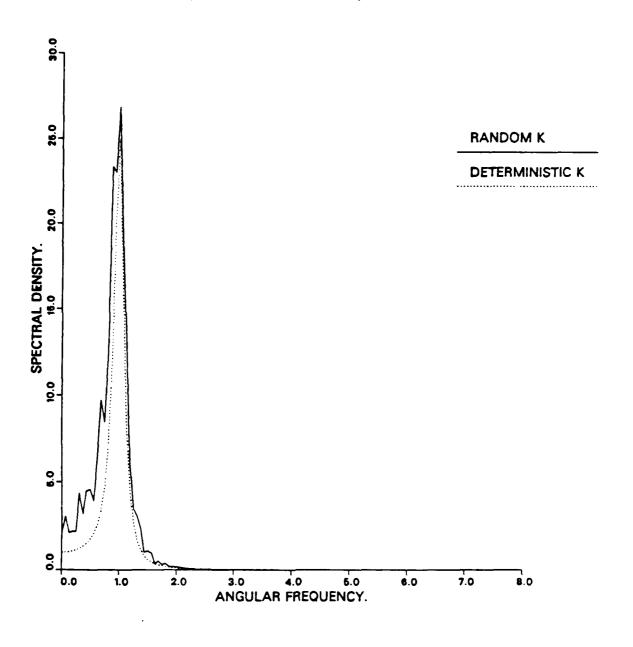
VARIANCE, DETERMINISTIC = 2.50, RANDOM =23.31





VARIANCE OF K = 0.20, BETA = 1.00, ALPHA =10.00

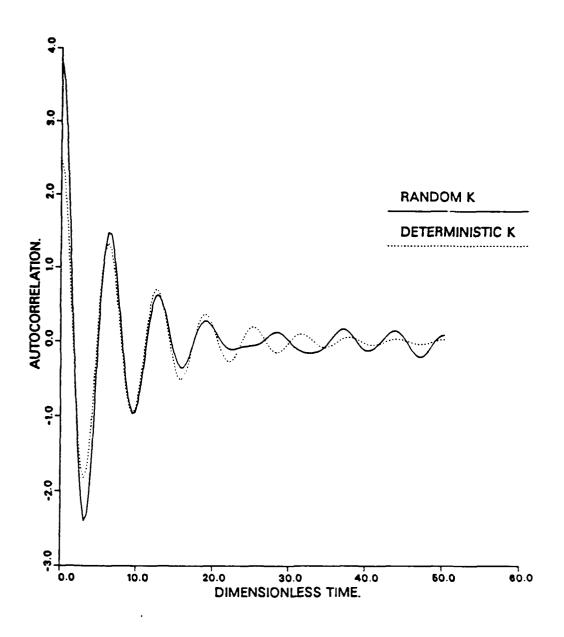
PEAK AT 1.0, DETERMINISTIC =25.00, RANDOM =26.82





VARIANCE OF K = 0.20, BETA = 1.00, ALPHA =10.00

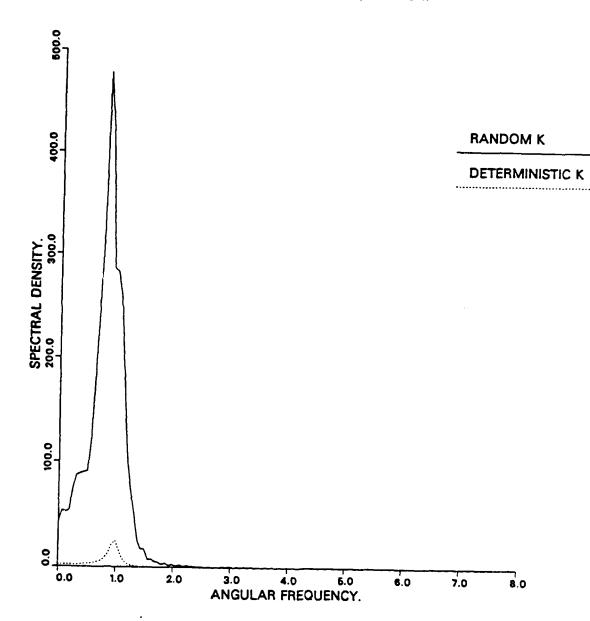
VARIANCE, DETERMINISTIC = 2.50, RANDOM = 3.86





VARIANCE OF K = 0.50, BETA = 1.00, ALPHA = 1.00

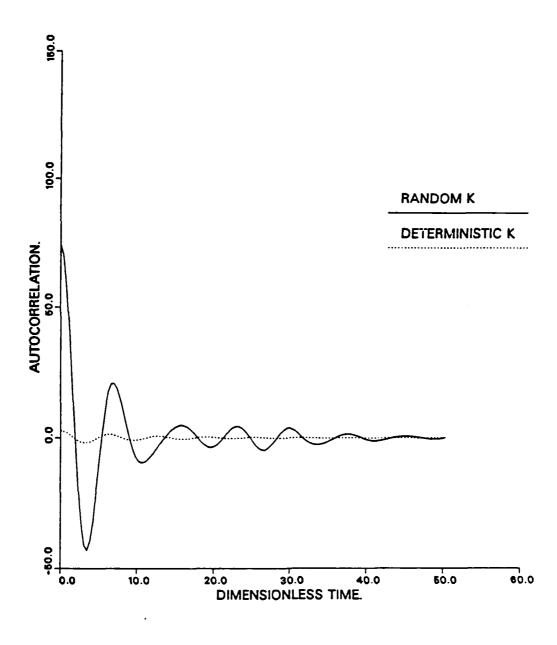
PEAK AT 1.0, DETERMINISTIC =25.00 , RANDOM =\*\*\*\*\*





VARIANCE OF K = 0.50, BETA = 1.00, ALPHA = 1.00

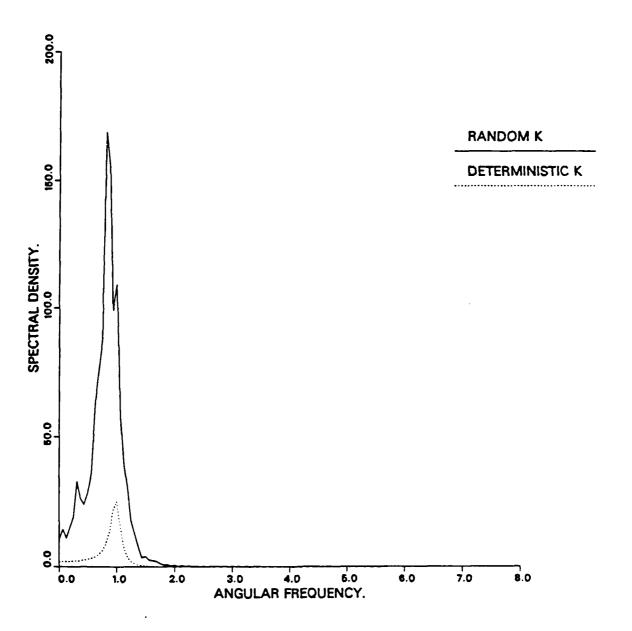
VARIANCE, DETERMINISTIC = 2.73, RANDOM =74.38





VARIANCE OF K = 0.40, BETA = 1.00, ALPHA = 1.00

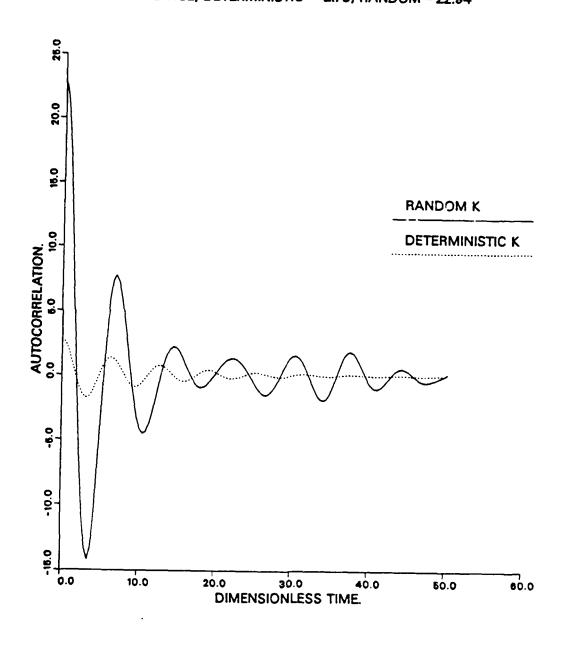
PEAK AT 1.0, DETERMINISTIC =25.00, RANDOM =\*\*\*\*\*



RANDOM VS. DETERMINISTIC SYSTEM.

VARIANCE OF K = 0.40, BETA = 1.00, ALPHA = 1.00

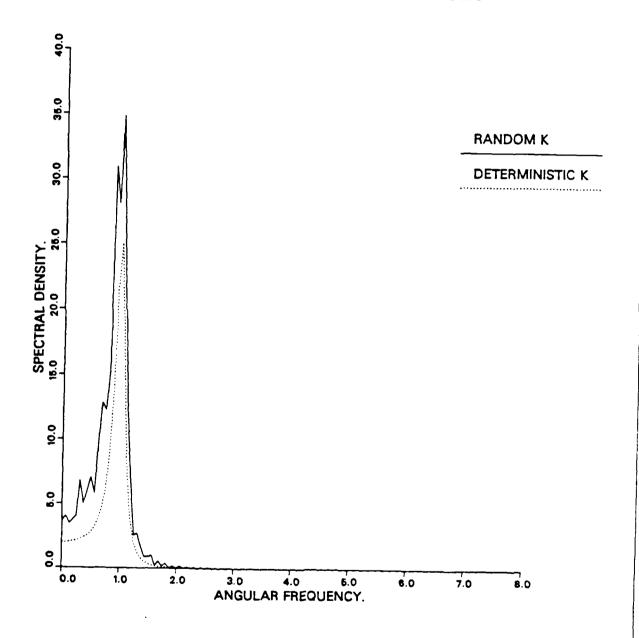
VARIANCE, DETERMINISTIC = 2.73, RANDOM = 22.94

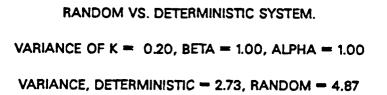


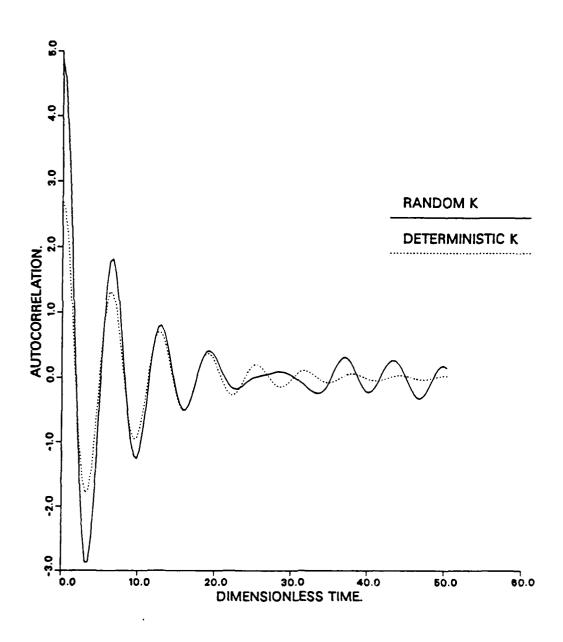


VARIANCE OF K = 0.20, BETA = 1.00, ALPHA = 1.00

PEAK AT 1.0, DETERMINISTIC =25.00 , RANDOM =34.78



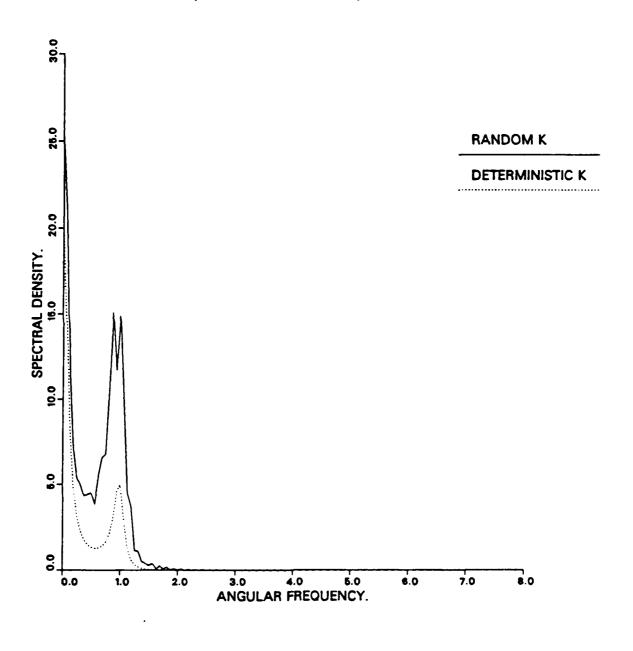


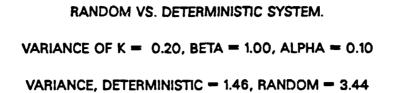


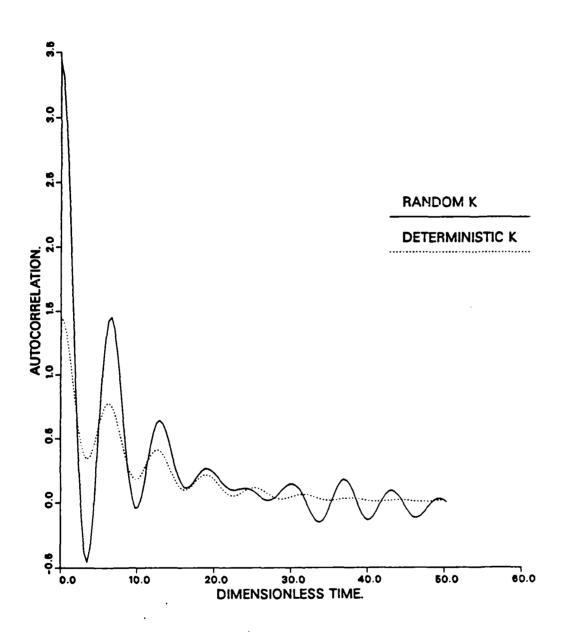


VARIANCE OF K = 0.20, BETA = 1.00, ALPHA = 0.10

PEAK AT 1.0, DETERMINISTIC = 4.95, RANDOM =14.86



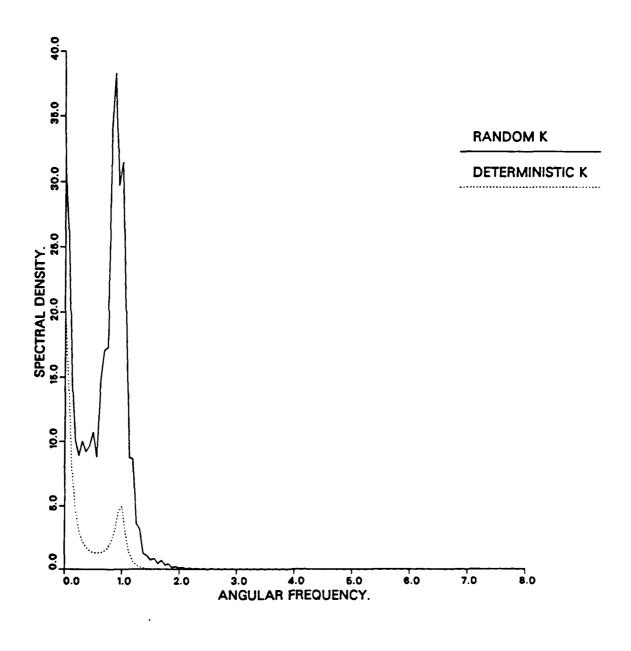






VARIANCE OF K = 0.30, BETA = 1.00, ALPHA = 0.10

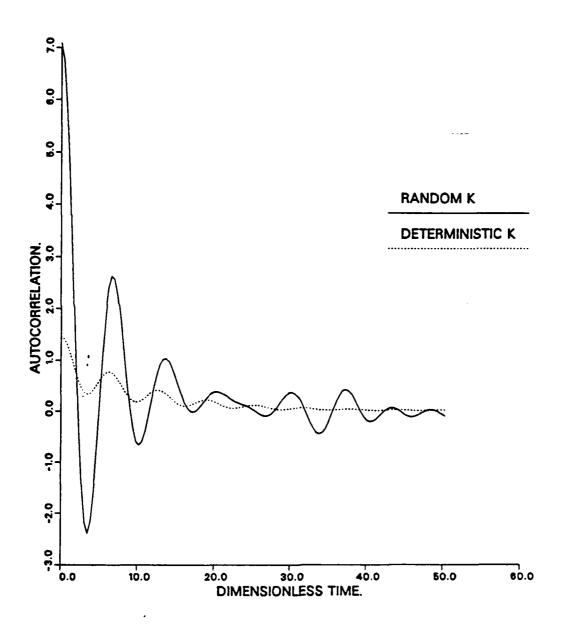






VARIANCE OF K = 0.30, BETA = 1.00, ALPHA = 0.10

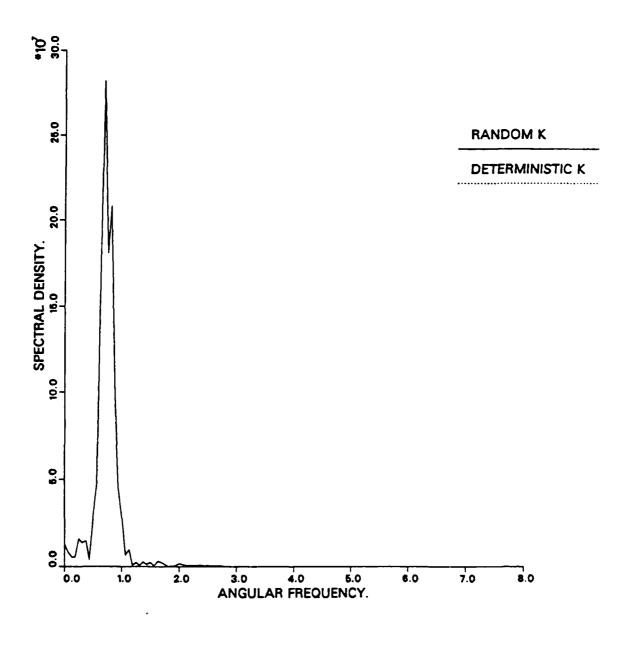
VARIANCE, DETERMINISTIC = 1.46, RANDOM = 7.09

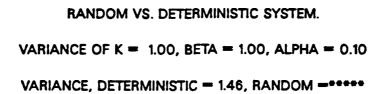


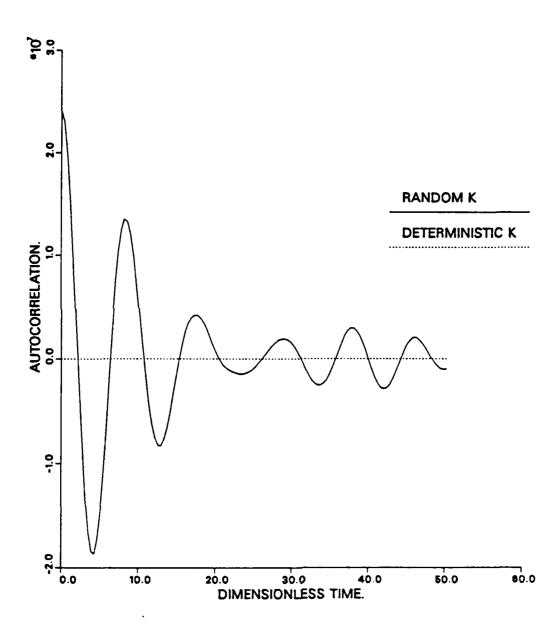


VARIANCE OF K = 1.00, BETA = 1.00, ALPHA = 0.10

PEAK AT 1.0, DETERMINISTIC = 4.95, RANDOM =\*\*\*\*\*



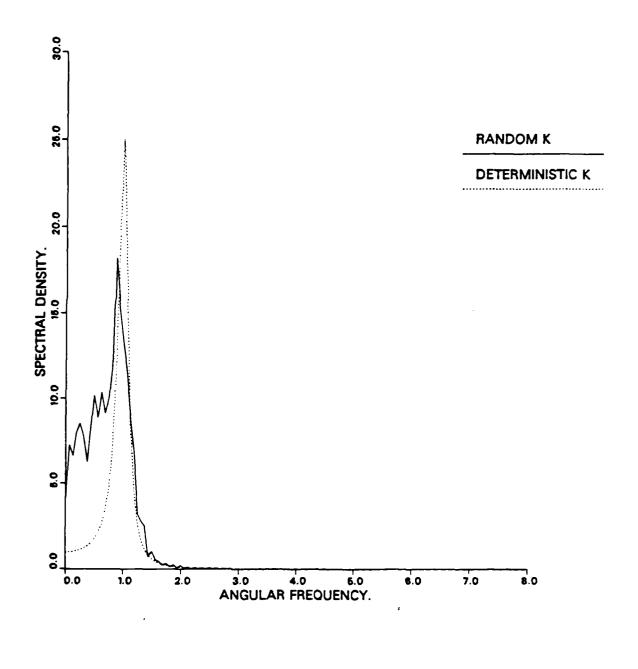


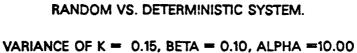




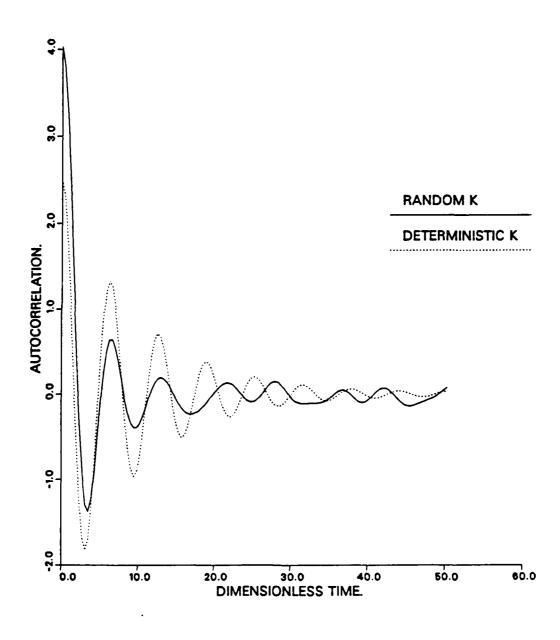
VARIANCE OF K = 0.15, BETA = 0.10, ALPHA = 10.00

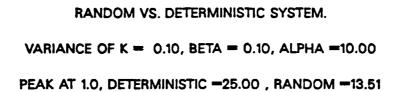
PEAK AT 1.0, DETERMINISTIC =25.00 , RANDOM =13.33

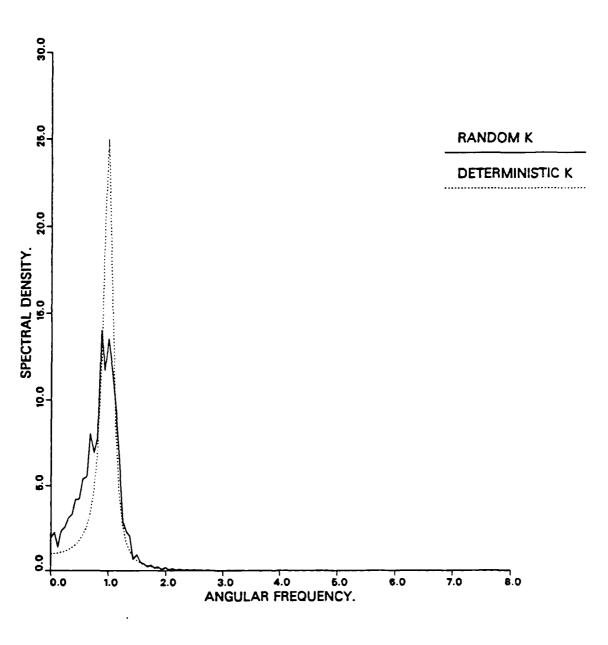




VARIANCE, DETERMINISTIC = 2.50, RANDOM = 4.03

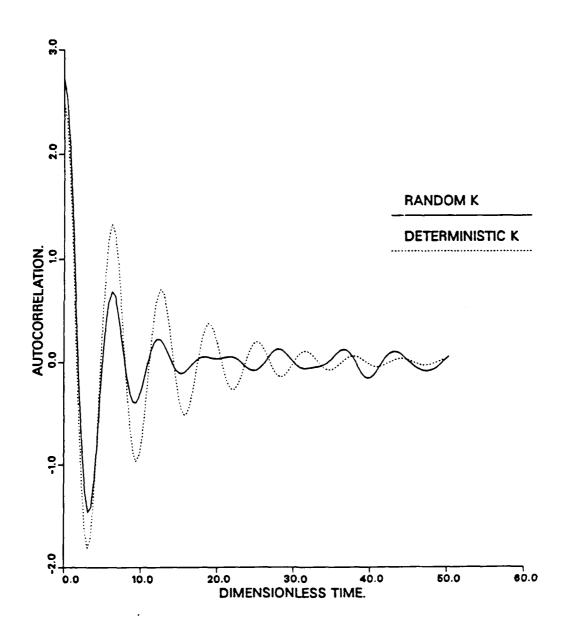






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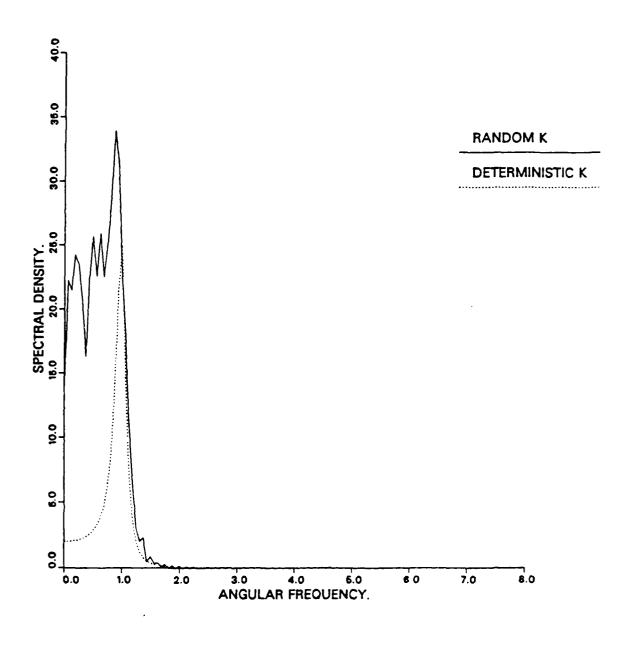






VARIANCE OF K = 0.15, BETA = 0.10, ALPHA = 1.00

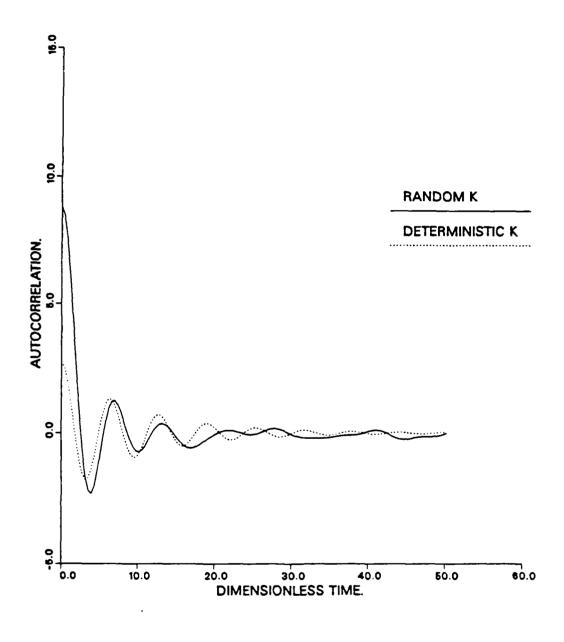
PEAK AT 1.0, DETERMINISTIC =25.00, RANDOM =23.01





VARIANCE OF K = 0.15, BETA = 0.10, ALPHA = 1.00

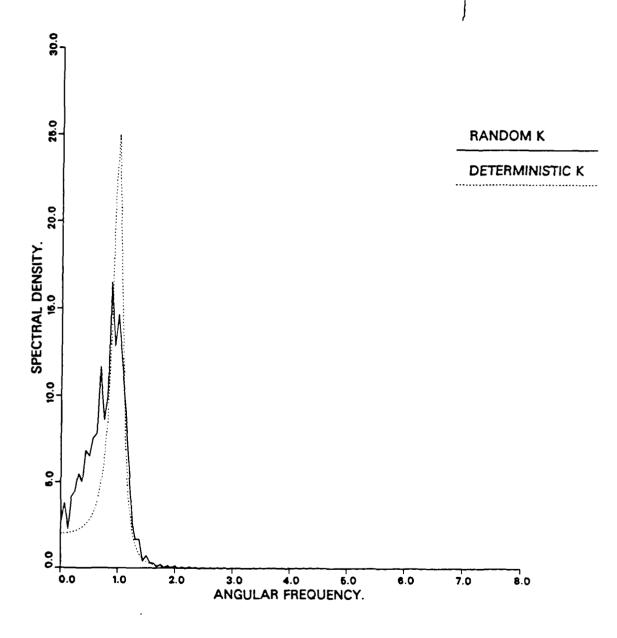
VARIANCE, DETERMINISTIC - 2.73, RANDOM - 8.80

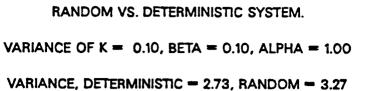


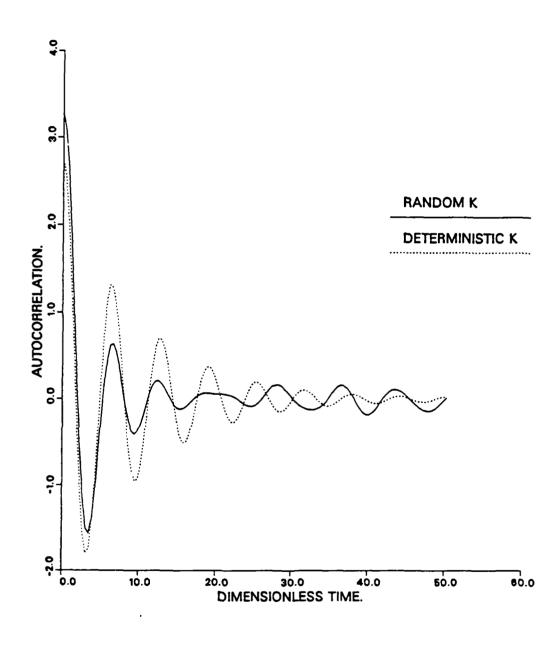


VARIANCE OF K = 0.10, BETA = 0.10, ALPHA = 1.00

PEAK AT 1.0, DETERMINISTIC -25.00 , RANDOM -14.62



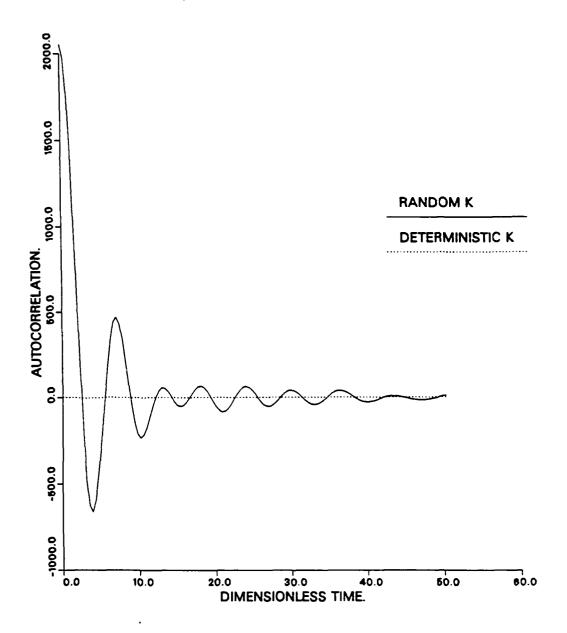






VARIANCE OF K = 0.20, BETA = 0.10, ALPHA = 1.00

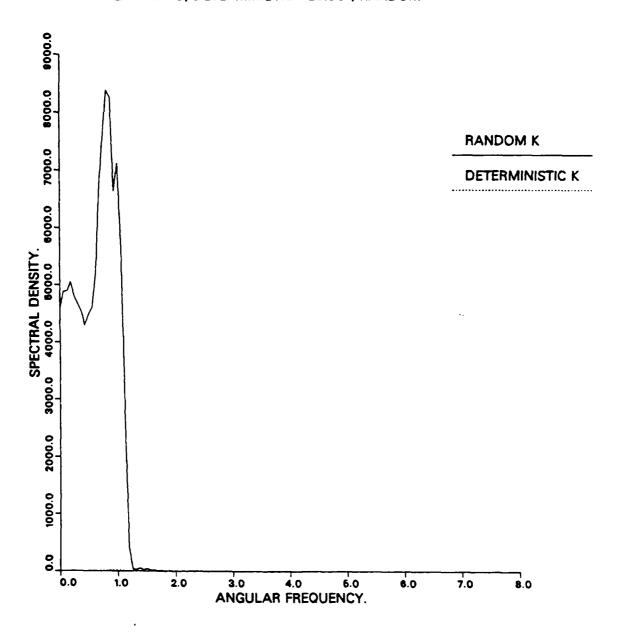
VARIANCE, DETERMINISTIC = 2.73, RANDOM =\*\*\*\*\*





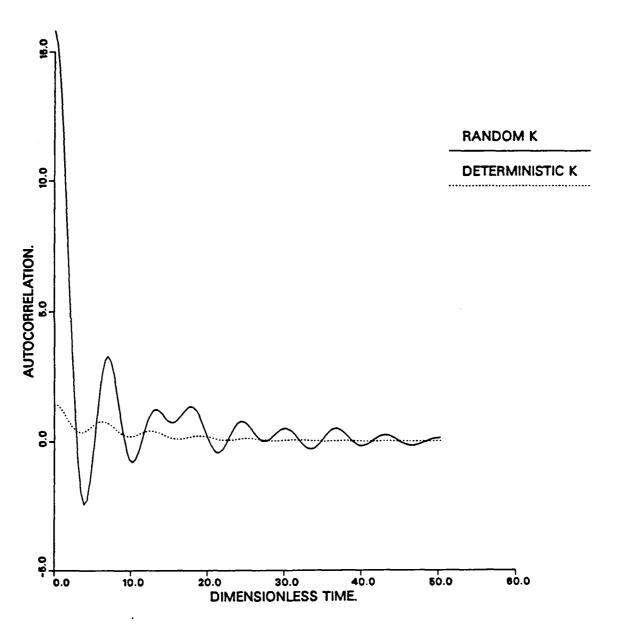
VARIANCE OF K = 0.20, BETA = 0.10, ALPHA = 1.00

PEAK AT 1.0, DETERMINISTIC =25.00 , RANDOM =\*\*\*\*\*



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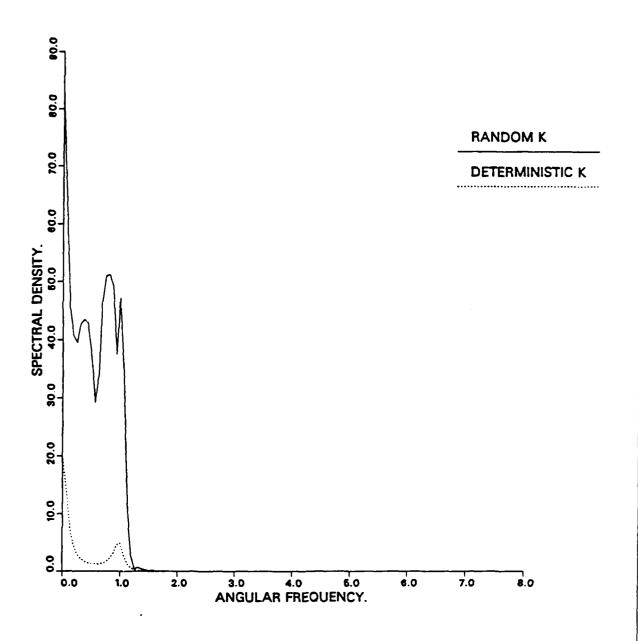






VARIANCE OF K = 0.15 BETA = 0.10 ALPHA = 0.10

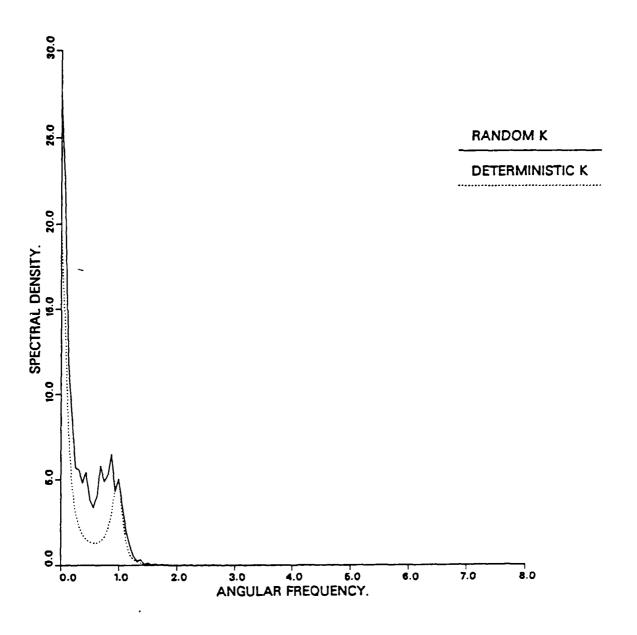
PEAK AT 1.0, DETERMINISTIC = 4.95 RANDOM =47.12

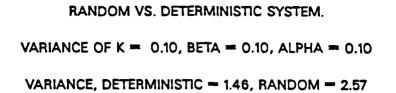


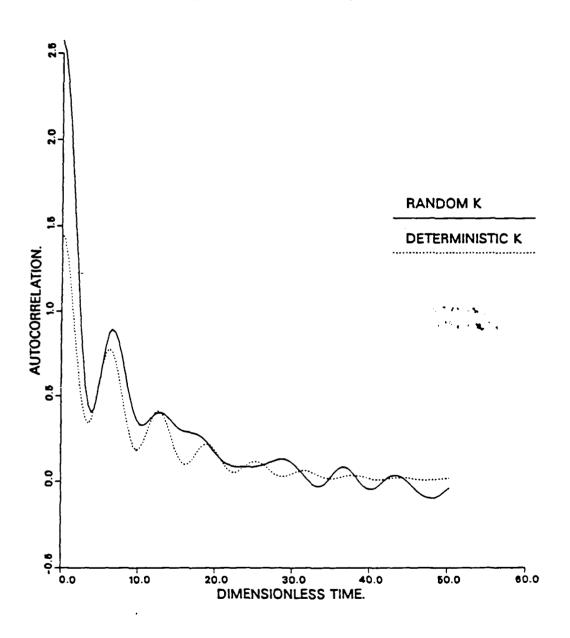


VARIANCE OF K = 0.10, BETA = 0.10, ALPHA = 0.10

PEAK AT 1.0, DETERMINISTIC = 4.95, RANDOM = 5.02







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